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2017 Rep. Prog. Phys. 80 022001

(http://iopscience.iop.org/0034-4885/80/2/022001)

IOP Publishing
Reports on Progress in Physics
Rep. Prog. Phys. **80** (2017) 022001 (10pp)
doi:10.1088/1361-6633/aa50e4

Key Issues Review

A brief history of the multiverse

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Received 4 October 2016, revised 28 October 2016
Accepted for publication 30 November 2016
Published 10 January 2017

Corresponding Editor Professor Leszek Roszkowski

Abstract
The theory of the inflationary multiverse changes the way we think about our place in the world. According to its most popular version, our world may consist of infinitely many exponentially large parts, exhibiting different sets of low-energy laws of physics. Since these parts are extremely large, the interior of each of them behaves as if it were a separate universe, practically unaffected by the rest of the world. This picture, combined with the theory of eternal inflation and anthropic considerations, may help to solve many difficult problems of modern physics, including the cosmological constant problem. In this article I will briefly describe this theory and provide links to the some hard to find papers written during the first few years of the development of the inflationary multiverse scenario.

Keywords: inflationary cosmology, chaotic inflation, eternal inflation, multiverse, cosmological constant, anthropic principle, string theory landscape

(Some figures may appear in colour only in the online journal)

1. Introduction

The theory of inflationary multiverse is based on unification of inflationary cosmology, anthropic considerations, and particle physics. Its most advanced versions include a combination of eternal inflation and string theory into what is now called ‘string theory landscape’. This theory is still ‘work in progress’, and the pendulum of public opinion with respect to it swings with a very large amplitude. Some people love this theory, others hate it and write papers defending the integrity of physics. It does not help much that the word ‘multiverse’ is used differently by different people. In this situation it may be useful to remember what exactly we are talking about and why this theory was invented.

This is not an easy task. Until the mid-90’s the cosmological anthropic principle pioneered by Dicke, Carter, Rees, Barrow, Rozental and others remained very unpopular. For example, we know that the proton mass is almost exactly equal to the neutron mass. If the proton were 1% heavier or lighter, life as we know it would be impossible. Similarly, one cannot significantly change the electron charge and its mass without making the universe unsuitable for life. But if physical parameters are just constants in the Lagrangian, nothing can change them. Therefore the standard lore was that one should avoid using anthropic arguments for explaining fundamental properties of our world.

As a result, some of the key ideas relating to each other inflation and anthropic considerations originally were expressed in a rather cryptic form and scattered among preprints, conference proceedings and old journals which are hard to find. In this paper I will briefly describe the history of the evolution of these ideas and provide links to some of the original publications which are especially difficult to find.

Historically, there were many different versions of the theory of the multiverse based on the many-world interpretation of quantum mechanics [1] and quantum cosmology [2], on the theory of creation of the universe ‘from nothing’ [3], and on the investigation of the Hartle–Hawking wave function [4]. These ideas are most powerful, but their consistent implementation requires deep understanding of difficult conceptual issues of quantum cosmology. Moreover, quantum cosmology by itself does not allow us to change fundamental constants.

Therefore the main progress in the development of the theory of the inflationary multiverse was achieved in a different, conceptually simpler context. To explain it, let us remember that one of the starting points of the pre-inflationary cosmology was that the universe is globally uniform. This was the so-called 'cosmological principle', which was invoked by many people, from Newton to Einstein, to account for the observed large-scale homogeneity of the universe. The physical mechanism explaining the homogeneity of our part of the world was provided by inflationary theory. Surprisingly enough, this theory made the cosmological principle obsolete.

Indeed, the main idea of inflationary cosmology is to make our part of the universe homogeneous by stretching any pre-existing inhomogeneities and by placing all possible 'defects', such as domain walls and monopoles, far away from us, thus rendering them unobservable. If the universe consists of different parts, each of these parts after inflation may become locally homogeneous and so large that its inhabitants will not see other parts of the universe, so they may conclude, incorrectly, that the universe looks the same everywhere. However, properties of different parts of the universe may be dramatically different. In this sense, the universe effectively becomes a multiverse consisting of different exponentially large locally homogeneous parts with different properties. To distinguish these exponentially large parts of our world from more speculative 'other universes' entirely disconnected from each other, I called these parts 'mini-universes', others call them 'pocket universes'. Eventually, we started using the word 'multiverse', or 'inflationary multiverse' to describe the world consisting of many different 'mini-universes', or 'pocket universes'.

An advanced version of this scenario describes our world as an eternally growing self-reproducing fractal consisting of many locally homogeneous parts (mini-universes). If the fundamental theory of all interactions has many different vacuum states, or allows different types of compactification, the laws of the low-energy physics and even the dimensionality of space in each of these mini-universes may be different. This provided, for the first time, a simple scientific interpretation of the anthropic principle, which did not rely on the possible existence of 'other universes': we can live only in those parts of the world which can support life as we know it, so there is a correlation between our own properties, and the properties of the part of the world that we can observe.

2. Inflationary multiverse: the first models

This combination of eternal inflation and the anthropic principle, which we will concentrate upon in this paper, was first proposed in 1982 in my preprint written during the famous Nuffield Symposium on inflationary cosmology [5]. It was argued there that an eternally inflating universe 'contains an infinite number of mini-universes (bubbles) of different sizes, and in each of these universes the masses of particles, coupling constants etc, may be different due to the possibility of different symmetry breaking patterns inside different bubbles. This may give us a possible basis for some kind of weak anthropic principle: there is an infinite number of causally unconnected mini-universes inside our universe, and life exists only in sufficiently suitable ones'. A full text of the preprint can be found at `www.stanford.edu/~alinde/1982.pdf`.

This idea, which plays the central role in the theory of inflationary multiverse, was further extended in my contribution to the proceedings of the Nuffield Symposium [6]. In addition to the discussion of various types of symmetry breaking in different parts of the multiverse, I also revisited there an old observation by Paul Ehrenfest that life as we know it may exist only in a four-dimensional space-time, because planetary and atomic systems would be unstable for $d \neq 4$ [7]. I argued in [6] that this idea finally acquires a well defined physical meaning in inflationary theory combined with the idea of spontaneous compactification: 'In the context of this scenario it would be sufficient that the compactification to the space $d = 4$ is *possible*, but there is no need for the four-dimensional space to be *the only* possible space after the compactification. Indeed, if the compactification to the space $d = 4$ is possible, there will be infinitely many mini-universes with $d = 4$ in which intelligent life can exist'.

Being impressed by free lunches provided to the participants of the Nuffield Symposium, I summarized the main consequence of this scenario as follows: 'As was claimed by Guth (1982), the inflationary universe is the only example of a free lunch (all matter in this scenario is created from the unstable vacuum). Now we can add that the inflationary universe is the only lunch at which all possible dishes are available' [6], see `www.stanford.edu/~alinde/LindeNuffield.pdf`.

Linde *et al* [5] described eternal inflation in a particular version of the new inflationary scenario [8] where the inflaton potential has a shallow minimum at the top. The regime of eternal inflation occurs in the old inflationary theory as well, but there it was considered a major obstacle precluding a consistent realization of inflationary cosmology [9, 10]. A possible existence of this regime in new inflation was first mentioned by Steinhardt in his talk at the Nuffield symposium. A brief discussion of this idea, as well as of my paper [5], is contained in his contribution to the Proceedings of the Nuffield Symposium [11]. However, he did not discuss the use of the anthropic principle; in fact he opposes the theory of inflationary multiverse in strongest possible terms [12]. A more general approach to the theory of eternal inflation in the new inflationary scenario was developed in 1983 by Vilenkin [13]. He explained that inflationary quantum fluctuations may keep the field at the top of the potential in some parts of the universe even if the potential does not have a local minimum at the top.

The first practical application of the anthropic principle in the context of inflationary cosmology was given in [14]. It was shown there that inflationary fluctuations can induce transitions between different vacua in supersymmetric grand unified theories. These transitions divide the universe into exponentially large parts with different types of symmetry breaking, and therefore with different laws of low-energy physics. If not for this mechanism, the hot universe would stay forever in the SU(5) symmetric vacuum, unsuitable for life as we know it.

None of these developments attracted much attention at that time, in part because the anthropic principle was very unpopular, and in part because the new inflationary scenario did not work and was replaced by the chaotic inflation scenario [15]. The simplest versions of the chaotic inflation scenario describe inflation far away from any extrema of the inflaton potential, so the previous results obtained in [5, 11, 13] did not apply to it. In addition, the energy scale of eternal inflation in the new inflation scenario was extremely small as compared to the Planck scale. Therefore it was not clear whether inflationary fluctuations could be powerful enough to probe the inner structure of space and divide the universe into different parts with different types of compactification, as proposed in [6]. This problem was solved in 1986 with the discovery of the regime of eternal inflation in the chaotic inflation scenario [16, 17].

3. Eternal chaotic inflation

This proposal was initiated by the discovery of the regime of eternal inflation in the chaotic inflation scenario [16, 17]. (In fact, the name 'eternal inflation' was first introduced in these two papers.) It was shown there that eternal inflation is a generic regime which is possible in any theory with a sufficiently flat potential. This included not only new inflation, but any chaotic inflation model with polynomial potentials. More generally, eternal inflation occurs in any inflationary regime where the amplitude of density perturbations may become $O(1)$ for some values of the fields. The theory of this effect is briefly described in appendix A of this paper.

One of the most important features of this regime was the possibility of eternal inflation at arbitrary values of energy density, up to the Planck energy density. This implied that inflationary perturbations of all other fields produced at that epoch could be large enough to bring us from one vacuum state to another, and even from one type of compactification to another, practically irrespectively of the size of the barrier separating different vacua. This realization was one of the strongest emotional shocks of my life, which forced me to delay and completely re-write my book on inflationary cosmology, which I was preparing at that time [18].

First of all, the universe in this scenario becomes immortal. Not only that, but in the eternal process of self-reproduction it re-creates itself in all possible ways, probing all laws of low-energy physics compatible with the underlying physical theory. This opens the possibility to implement the anthropic principle in its strongest form, using the analogy with mutations of the laws of physics and the Darwinian approach [19]. The main idea was illustrated in [19] by an image of a multicolored self-reproducing universe, which gradually became the standard symbol of the inflationary multiverse. Figure 1 shows its version published in [19], with the original figure caption. By looking at this figure, one can instantly realize that eternal inflation transforms the universe into a huge eternally growing fractal. These are not just words: in 1987 Aryal and Vilenkin calculated fractal dimension of the universe in the new inflation scenario [20], and then, few years later, a similar calculation was performed for the eternal chaotic inflation [21].

I anticipated that the most interesting consequences of this scenario will be achieved in string theory and suggested that 'an enormously large number of possible types of compactification which exist e.g. in the theories of superstrings should be considered not as a difficulty but as a virtue of these theories, since it increases the probability of the existence of mini-universes in which life of our type may appear' [17]. But time was not yet ripe for eternal inflation in string theory because we did not know how to stabilize multiple string theory vacua discussed e.g. in [22]. The emphasis shifted to other aspects of the inflationary multiverse scenario. In particular, in order to make the anthropic selection viable, it was necessary to find physical mechanisms that would allow physical parameters to take different values in the context of the same physical theory. And here inflation helped to reveal many new possibilities.

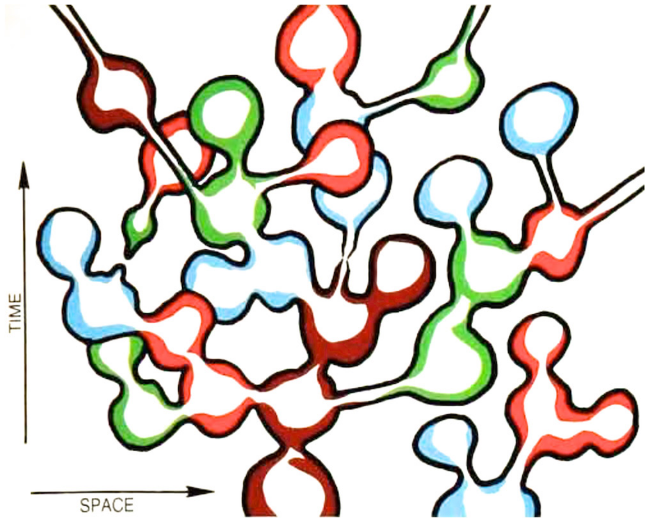

Figure 1. Global structure of a chaotic, self-reproducing inflationary universe. Locally (out to the 10^{10} light-year horizon) the universe looks quite homogeneous, but its global structure is complex. Mini-universes at the Planck energy density are 'mutants' that may forget completely the 'genetic code' (color) of their parent universe. They may even have a different space-time dimensionality. The typical thickness of a tube connecting two mini- universes after inflation is exponentially large, but if it corresponds to a compactified inflationary universe it can be as thin as the Planck length ($\sim 10^{-33}$ cm). If the tube then evaporates by Hawking radiation, the parent and offspring mini-universes have lost their umbilical space-time connection. (Reproduced from my paper [19], with the permission of the American Institute of Physics.)

For example, it was found that in certain mechanisms of baryogenesis, the ratio n_b/n_γ was controlled by a light scalar field which experienced quantum fluctuations during inflation [23]. As a result, the parameter n_b/n_γ in this scenario may take different values in different parts of the universe [24]. But why do we live in the part of the universe where n_b/n_γ is extremely small? A possible answer was that the large scale structure of the universe in its parts with relatively large ratio n_b/n_γ would be very different, which would make emergence of life as we know it much less likely. A similar argument was used in [25] to remove the standard constraints on the axion mass and explain the large ratio of dark matter to normal matter. Recently these ideas were revived and developed by several authors, see e.g. [26–28]. Investigation of inflationary perturbations in the Brans–Dicke theory has shown that the effective gravitational constant and the amplitude of perturbations of metric can take different values in different part of the post-inflationary universe [29]. Linde *et al* [30] contained an explicit realization of the possibility that our world may consist of different parts with different number of large (uncompactified) dimensions. Thus it was realized that many features of our world may indeed have anthropic origin. Most of these results were summarized in 1990 in my books 'Particle physics and inflationary cosmology' [18] and 'Inflation and quantum cosmology' [31].

The first ten years of the journey through the multiverse were very exciting but rather lonely. Not many people were interested in this theory; almost all of my papers on inflationary multiverse have been written without any collaborators. The editor of my book [18] recommended me to delete the part on the anthropic principle, saying that otherwise I will lose respect of my colleagues. I replied that if I do it, I will lose self-respect, and eventually she agreed to keep it. It appears though that the risk was very minimal anyway, since this part was in the very end of the book, in the chapter on quantum cosmology, which was of little interest to most of the readers at that time.

In the beginning of the 90's the situation began to change. In 1993 a detailed theory describing two different probability measures in the eternally inflating universe was proposed in a series of my papers written in collaboration with Arthur Mezhlumian, Juan Garcia-Bellido and Dimitri Linde [21, 32]. In 1994 Alex Vilenkin resumed his investigation of eternal inflation. His prolific work, as well as the work of his numerous talented collaborators, invigorated the theory of inflationary multiverse, bringing to it new interesting choices of the probability measure and anthropic considerations, see e.g. [33] and his book 'Many worlds in one' [34]. But despite this exciting progress, the theory of inflationary multiverse was developed only by a small group of experts and remained of no interest for the general physical community. And the crucial part of this theory, the anthropic considerations, was almost universally despised. The situation changed dramatically only after the discovery of the cosmological constant/dark energy and the development of the string theory landscape.

4. Multiverse, string theory landscape, and the cosmological constant problem

The discovery of dark energy in 1998 [35, 36] pushed the cosmological constant problem to the forefront of research. The observers found that empty space is not entirely empty, it has tiny energy density $\sim 10^{-29}$ g cm^{-3}. This minuscule number is 120 orders of magnitude smaller than the Planck density, it is more than 40 orders of magnitude smaller than nuclear density and 29 orders of magnitude smaller than density of water. In the early universe, the energy density of usual matter was many orders of magnitude greater than $\sim 10^{-29}$ g cm^{-3}; in the distant future it will be much smaller than the vacuum energy density. But for some mysterious reason it is of the same order of magnitude than the energy density of matter of the universe at present time. This discovery triggered an unexpected chain of events in theoretical physics.

For many decades theorists were unsuccessfully trying to find a theory which would explain why the cosmological constant, representing the vacuum energy density, is exactly zero. But we could not do it; it was a spectacular failure. After 1998, we faced a much more complicated problem: it was necessary to explain why vacuum energy/cosmological constant is not exactly zero but is extremely small, and why this constant is of the same order of magnitude as the density of normal matter at the present epoch. The only known way to address these two problems without invoking incredible fine-tuning was related to the anthropic principle, and, therefore, to the theory of the multiverse.

The possibility that the value of the vacuum energy may be determined by the anthropic considerations was known long ago. For example, back in 1981 Davies and Unwin [37] noted: 'In the absence of a fundamental reason why Λ should be so small, a possible anthropic explanation suggests itself. Perhaps the excessive smallness of Λ is a feature that only characterizes our particular region of the universe. In other regions this fine-tuning fails and Λ assumes much greater values. But in such regions Λ would dominate the gravitational dynamics, leading to exponential expansion, or (for negative values) collapse into anti-de Sitter space. Probably, values that differ by more than a few orders of magnitude from the observed upper limit in our region would be sufficient to prevent the formation of galaxies, and hence organic life'. Similar arguments were given in [38–42]. But the real derivation of the anthropic bound on the positive cosmological constant was given for the first time by Weinberg in his famous paper of 1987 [43], and it was further strengthened in [44] and several subsequent publications.

The remaining problem was to find a physical mechanism that would allow the cosmological constant to vary and to become extraordinarily small. Even the first part of this problem was rather difficult because the cosmological constant was considered by many to be just a constant in the gravitational action, so it was not quite clear whether it makes any sense to consider universes with different values of Λ. Then in 1974 it was realized that a constant scalar field may play the role of a cosmological constant, taking different values in different places at different time [45]. But having 10 or 100 different vacua with different values of Λ would not explain why it is as small as 10^{-120} in Planck units. To address this issue, Davies and Unwin [37] suggested to use a twisted configuration of the scalar field interpolating between two minima of the Higgs-type potential, in a hope to represent different values of the effective cosmological constant by the slowly changing spatial distribution of a scalar field. However, for the parameters used in [37] this distribution represented a narrow domain wall, which did not lead to the cancellation of Λ. In an attempt to explain the smallness of the cosmological constant, Banks introduced one of the first models of dark energy, but after studying it he concluded [41]: 'This is a deadblow for the present model. Even the anthropic principle cannot save it'. In what follows I will describe three different ideas proposed in 1984–1986 [38–40], which found their way to modern discussions of the cosmological constant problem.

(1) The first of these proposals [38] was based on quantum cosmology and the possibility of creation of the universe 'from nothing'. Initially, papers on quantum creation of the universe [3, 4] suggested that the probability of creation of a closed inflationary universe with the inflaton energy density $V(\phi)$ is given by $\mathrm{e}^{24\pi^2/V(\phi)}$. This implied that the probability of quantum creation of an inflationary universe is exponentially small, suggesting that it is much easier to create a huge empty universe of a present size, with a tiny value of the cosmological constant. However, the subsequent investigation indicated that the probability of the universe creation is in fact given by $\mathrm{e}^{-24\pi^2/V(\phi)}$ [38, 46, 47]. This provided an ideal framework for realization of initial conditions for chaotic inflation, where V can be large [38, 46]. There could be a problem for those models where inflation is possible only for $V \ll 1$ and the term $\mathrm{e}^{-24\pi^2/V(\phi)}$ is exponentially small, but one could alleviate this problem using anthropic considerations [47]. Moreover, an investigation of quantum creation of compact open or flat universes with nontrivial topology indicated that this process is not exponentially suppressed even for $V \ll 1$ [48–50]. And once inflation starts, its nontrivial topology becomes practically unobservable: the universe becomes locally uniform and isotropic.

In order to use these results for solving the cosmological constant problem, one may consider a combined contribution of scalar fields ϕ and fluxes (antisymmetric tensor fields $F_{\mu\nu\lambda\sigma}$) to vacuum energy. At the classical level the fields $F_{\mu\nu\lambda\sigma}$ take constant values all over the universe and contribute to the cosmological constant [51]. At that time it was not known that these fields may change their values due to tunneling with bubble formation [52]. Therefore one could expect that the cosmological constant is just a constant which cannot change. However, in quantum cosmology this is not necessarily an obstacle. One may study the probability of creation of different universes with different values of ϕ and $F_{\mu\nu\lambda\sigma}$. If one uses the expression for the probability $\mathrm{e}^{24\pi^2/V}$ [3, 4], one could conclude that Λ must vanish [51], which is not the case according to the observational data. However, if one uses the expression $\mathrm{e}^{-24\pi^2/V}$ [38, 46, 47], where $V = V(\phi) + V(F)$, one finds that the probability to live in a universe with a given value of Λ has a flat distribution as a function of Λ, which is exactly what we need for the anthropic solution of the cosmological constant problem, see the very end of the paper [38].

(2) The second mechanism was proposed by Sakharov [40]. He mentioned, following [6], that the universe may consist of many different parts with different types of compactification. Then he argued that if the number of compactified dimensions is sufficiently large, the number of different types of compactifications can be exponentially large. He emphasized that if this number is large enough, the typical energy gap between different levels can be extremely small, which may allow to explain the smallness of the cosmological constant by using anthropic considerations, see section 4 in `www.stanford.edu/~alinde/Sakharov1984.pdf`.

(3) The third mechanism was proposed in [39]. It was based on a combination of eternal inflation driven by the inflaton field ϕ and a subsequent slow roll of what was later called 'quintessence' field Φ. The role of eternal inflation was to generate perturbations of the field Φ, which then give this field different values in different exponentially large parts of the universe. As a result, the universe becomes divided into different parts with a flat probability distribution for different values of the effective cosmological constant. Once again, this provided a possibility to use anthropic considerations for solving the cosmological constant problem, see `www.stanford.edu/~alinde/1986300yrsgrav.pdf`.

All of these ideas did not attract much interest until the discovery of the cosmological constant in 1998. This discovery created a lot of tension, especially since the cosmological constant was positive, and at that time we did not know any way of describing a positive cosmological constant in string theory.

An important step was made in 2000 by Bousso and Polchinski [53], who proposed a string theory motivated model explaining a possible reason for an exponentially large number of vacua with different values of the cosmological constant. This could provide an anthropic solution to the cosmological constant problem. The idea was brilliant, preserving some elements of the suggestions made in [38, 40], but going much further. But at that time we still did not know how one could consistently construct any stable or metastable string theory vacua with positive vacuum energy (de Sitter vacua).

Eventually, several interesting mechanisms of vacuum stabilization in string theory have been proposed, see in particular [54]. The most developed one, the so-called KKLT construction [55], was proposed in 2003. We found a possible way to stabilize string theory vacua, but we also found that all de Sitter vacua in string theory are not absolutely stable but metastable, and all barriers separating these vacua can be penetrated with finite probability. This was the crucial observation, which implied that the universe can exponentially expand in any of these metastable de Sitter states as in the eternal inflation scenario, and tunnel from each of them to any other string theory vacuum. I will describe the proof of this important statement in the appendix B. Estimates made in [56] have shown that the total number of such vacua can be as large as 10^{500}. Even if one starts with the universe in one of these vacua, a combination of eternal inflation and the inevitable tunneling between these vacua should create a multiverse consisting of parts where all of these de Sitter vacua are represented. Lenny Susskind called this scenario 'the string theory landscape' [57]. An incredible richness of this landscape may help us to solve the cosmological constant problem, as well as many other problems which required anthropic explanation. This perfectly matched the earlier expectations expressed in [17].

After these developments, the general attitude towards inflationary multiverse and anthropic considerations changed. Powerful efforts by Vilenkin, Susskind, Bousso, Guth, Shenker, Hall, Nomura and many others rapidly transformed this field into a vibrant and rapidly developing branch of theoretical physics. String theory underpinnings of vacuum stabilization, uplifting, inflation, and other aspects of the multiverse scenario received strong boost from subsequent works of Silverstein, Kachru, Polchinski, Kallosh and others; see especially the recent series of papers [58–62]. There is a broad agreement between different scientists about the general structure of the inflationary multiverse, and there are even some attempts to expand the concept of the multiverse from physics to mathematics [63]. But much more work is required if we want to use our new knowledge for making unambiguous scientific predictions and explaining our observations. Indeed, if eternal inflation produces infinitely many red universes and infinitely many green universes, then which of these infinite sets of universes will dominate? This is called the measure problem [21].

This problem is not specific to the multiverse, it is just a byproduct of dealing with infinities. Consider for example the usual flat or open Friedmann universe. It is infinite, and therefore it contains infinite number of planets, infinite number of cities, and infinite number of rooms. Since there are infinitely many of them, there are also infinitely many 'bad' rooms where all oxygen molecules simultaneously move into a corner of the room, and everybody suffocates. The probability of this process in each particular room is incredibly small, but if one takes into account that the number of 'bad' rooms is infinite, the statements about the probabilities become ambiguous. For example, one can take one 'bad' room and one 'normal' room, then yet another 'bad' and 'normal' room, continue this counting for indefinitely long time and conclude that 50% of all rooms are 'bad'.

This is one of the many paradoxes which appear when one compares infinities. One way to deal with it is to follow the lead of quantum mechanics and make only those predictions that an observer can actually verify, under initial conditions that he/she should know/prepare before making predictions. This makes the subset of accessible rooms finite, and the normal expectations based on the standard laws of thermodynamics prevail.

It is too early to tell which of the proposed solutions of the measure problem will be finally accepted; for some recent reviews and proposals see e.g. [64–67]. It is important that whereas different approaches lead to slightly different constraints on the possible values of the cosmological constant, these differences do not change the main qualitative conclusion: many parts of inflationary multiverse are expected to be in a state with a very large absolute value of the cosmological constant; we cannot live there. But there are many exponentially large parts of the multiverse with an extremely small value of the cosmological constant, compatible with our existence. That is all that we can say with reasonable certainty, and this is already quite sufficient to make the cosmological problem if not completely and unambiguously solved then at least significantly ameliorated.

Similarly, there is a strong correlation between life as we know it and the values of the electron charge and its mass, as well as with the ratio between the mass of the proton and the mass of the neutron, and with the number of non-compactified dimensions. Therefore some of the physical parameters describing our part of the world may be environmental, specific to the part of the multiverse where we can live. Meanwhile a discovery of many deep symmetries in the theory of elementary particles suggests that relations between many other parameters may be fundamental rather than environmental. This relates our discussion to the famous statement made by Einstein in this 'Autobiographical Notes' [68]:

> *I would like to state a theorem which at present can not be based upon anything more than a faith in the simplicity, i.e., intelligibility, of nature: There are no arbitrary constants..., nature is so constituted that it is possible logically to lay down such strongly determined laws that within these laws only rationally completely determined constants occur (not constants, therefore, whose numerical value could be changed without destroying the theory).*

The theory of inflationary multiverse does not challenge this philosophical attitude, which is deeply rooted in the basic principles of science. But what we have found is that the total number of truly fundamental constants may be much smaller than what one could expect by studying the observable part of the universe. Sometimes what we perceive as a fundamental constant may be just an environmental parameter, which seems constant because of the effects of inflation, but which may take entirely different values in other parts of the world.

An often expressed concern about this theory is that we may not see different parts of the multiverse any time soon, and until this happens there will be no experimental evidence supporting this theory. However, I do believe that we already have strong experimental evidence in favor of the theory of the multiverse.

In order to explain it, let us take a step back to the time when the inflationary theory was invented. Its main goal was to address many problems which at that time could seem rather metaphysical: why is our universe so big? Why is it so uniform? Why is it isotropic, why it does not rotate like our galaxy? Why parallel lines do not intersect? It took some time before we got used to the idea that the large size, flatness, isotropy and uniformity of the universe should not be dismissed as trivial facts of life. Instead of that, they should be considered as *experimental data* requiring an explanation, which was provided with the invention of inflation.

Similarly, the anomalously small value of the cosmological constant, the extreme smallness of the electron mass, the near coincidence between the proton and neutron masses, as well as the fact that we live in a 4-dimensional space, are *experimental data*, and the only presently available plausible explanation of these and many other surprising experimental results has been found within the general framework of the theory of the multiverse. And, talking about coincidences, even though possible roles of inflation and string theory in this construction have been conjectured 30 years ago [17], the way how different parts of the puzzle started falling into proper places within the context of the string theory landscape was nothing short of miraculous.

That was one of the main reasons why I decided to describe the history of the development of the theory of inflationary multiverse and discuss different pieces of hard to find old papers containing some early and often rather naive formulations of the future theory. Hopefully by looking at it one may either find a possibility to deviate from our original path, or understand why many of us take the admittedly incomplete progress towards the theory of the inflationary multiverse so seriously.

Acknowledgments

This work was supported by the SITP, by the NSF Grant PHY-1316699 and by the Templeton foundation grant 'Inflation, the Multiverse, and Holography.'

Appendix A. Eternal chaotic inflation

Chaotic inflation scenario describes the scalar field ϕ slowly rolling down to the minimum of its potential $V(\phi)$ during the nearly exponential expansion of the universe [15]. This regime occurs in a broad class of inflationary models with sufficiently flat potentials. It make the universe almost exactly homogeneous on the scale much greater than the typical cosmological scale H^{-1}, where $H = \dot{a}/a$, $a(t)$ is a scale factor of the universe. Equation of motion for a homogeneous canonically normalized scalar field in the slow-roll inflationary regime is

$$3H\dot{\phi} = -V'. \tag{A.1}$$

During inflation

$$H^2 = V(\phi)/3. \tag{A.2}$$

The universe in this regime expands nearly exponentially, $a(t) \approx \mathrm{e}^{H(t)t}$, where H does not change much during the cosmological time H^{-1}. Equations (A.1) and (A.2) imply that during the Hubble time $\delta t = H^{-1}$ the field ϕ decreases by

$$\Delta\phi = -V'/V. \tag{A.3}$$

This equation suggests that during inflation the field ϕ always rolls down towards the minimum of the potential. That is why the possibility that the field may eternally jump against the flow and climb higher and higher [16, 17] was so counter-intuitive.

In order to describe this process, one should take unto account that space is filled with quantum fluctuations of all types of physical fields. These fluctuations can be considered as waves of physical fields with all possible wavelengths, moving in all possible directions. The wavelengths of all vacuum fluctuations of the scalar field ϕ grow exponentially in the expanding universe. When the wavelength of any particular fluctuation becomes greater than H^{-1}, this fluctuation stops oscillating, and its amplitude freezes at some nonzero value $\delta\phi(x)$ because of the large friction term $3H\dot{\phi}$ in the equation of motion of the field ϕ. The amplitude of this fluctuation then remains almost unchanged for a very long time, whereas its wavelength grows exponentially.

The appearance of such a frozen fluctuation is equivalent to the appearance of a classical field $\delta\phi(x)$ that look homogeneous on the scale of the horizon H^{-1}. The average amplitude of such perturbations generated during a time interval H^{-1} is given by [18]

$$|\delta\phi(x)| \approx \frac{H}{2\pi}, \tag{A.4}$$

which is equal to the Hawking temperature in de Sitter space with the Hubble constant H. Each time interval H^{-1}, new fluctuations of this magnitude and wavelength H^{-1} appear and freeze on top of the previously frozen fluctuations.

Thus the evolution of the classical scalar field ϕ becomes similar to the Brownian motion: in average, each time H^{-1} the field ϕ slides down by $\Delta\phi = -V'/V$. However, in each part of the universe of a size H^{-1} (the size of the horizon) the field may additionally jump either up or down by $\delta\phi(x) \sim \frac{H}{2\pi} \sim \frac{\sqrt{V}}{2\sqrt{3}\pi}$. The ratio

$$\frac{\delta\phi(x)}{\Delta\phi} \sim \frac{V^{3/2}}{2\sqrt{3}\pi V'} \tag{A.5}$$

is proportional to the amplitude of *post-inflationary* scalar perturbations of the metric produced at that time [69, 70]. This ratio is very small during the last 60 e-foldings of inflation, but it may be very large in the very early universe.

For example, in the simplest model $V(\phi) = \frac{m^2\phi^2}{2}$ one has

$$\frac{\delta\phi(x)}{\Delta\phi} \sim \frac{m\phi^2}{4\sqrt{6}\pi}. \tag{A.6}$$

This means that for $\phi \gg m^{-1/2}$ one has $\delta\phi(x) \gg \Delta\phi$, so the amplitude of jumps up and down is much greater than the overall decrease of the field. Each time H^{-1} the size of the universe grows e times, and its volume grows $e^3 \sim 20$ times. For $\delta\phi(x) \gg \Delta\phi$, this means that the volume of the parts of the universe where the value of the field ϕ becomes *greater* than it was before grows 10 times within the time interval H^{-1}. And during the next time interval H^{-1}, the volume of the parts of the universe with a growing field ϕ will grow 10 times again. This leads to eternal inflation in the chaotic inflation scenario [16, 17].

Since the scalar perturbations of metric in a post-inflationary universe are proportional to $\frac{\delta\phi(x)}{\Delta\phi}$ (A.6), one could wonder whether our considerations concerning eternal inflation are reliable. However, scalar perturbations (A.6) become large only *after* inflation. Meanwhile during inflation, the perturbations of energy density during inflation are very small, of the order $V^2 \ll V$ for sub-Planckian values of the potential $V \ll 1$ [18, 69]. For example for $m \sim 10^{-5}$, eternal inflation becomes possible at $V(\phi) > 10^{-5}$, i.e. 5 orders of magnitude below the Planck density. The large amplitude of the scalar perturbations of metric in a post-inflationary universe is a direct consequence of eternal inflation: a sufficiently large part of the universe, which corresponds to inflation starting from $\phi \gg m^{-1/2}$, there will be parts of the universe where inflation is over, and other parts, where eternal inflation still continues. This simply means that the global structure of the universe cannot be described by the FRW metric: the universe becomes a fractal, see figure 1.

The conditions for eternal inflation in simplest inflationary models such as $V(\phi) = \frac{m^2\phi^2}{2}$ are satisfied all the way up to the Planck density. Close to the Planck density, the Hubble constant could be as large as the Planck mass, and inflationary fluctuations of all fields become maximally strong, experiencing Planck size jumps. This should allow these fields to easily jump from one vacuum state of the theory to another, which could lead not only to modifications in the low-energy laws of physics, but even to jumps between different types of compactification of space time, including transitions with different number of non-compactified dimensions [6, 16, 17, 30, 71].

Appendix B. A lower bound on the transition rate between different vacua and string theory landscape

Consider de Sitter vacuum state φ_0 corresponding to the local minimum of the potential with $V_0 > 0$. Suppose that there is another vacuum φ_1 separated from φ_0 by a barrier with $V(\phi) > 0$. What can we say about the possibility of tunneling between these vacua?

To describe tunneling from a local minimum at $\varphi = \varphi_0$ following Coleman and De Luccia [72] one should consider an $O(4)$-invariant Euclidean spacetime with the metric

$$\mathrm{d}s^2 = \mathrm{d}\tau^2 + b^2(\tau)(\mathrm{d}\psi^2 + \sin^2\psi\, \mathrm{d}\,\Omega_2^2). \tag{B.1}$$

The scalar field φ and the Euclidean scale factor (three-sphere radius) $b(\tau)$ obey the equations of motion

$$\varphi'' + 3\frac{b'}{b}\varphi' = V_{,\varphi}, \quad b'' = -\frac{b}{3}(\varphi'^2 + V), \tag{B.2}$$

where primes denote derivatives with respect to τ. (We use the system of units $M_p = 1$.)

Coleman–De Luccia instantons describe the field $\varphi(\tau)$ beginning in a vicinity of the false vacuum φ_0 at $\tau = 0$, and reaching some constant value $\varphi_f > \varphi_1$ at $\tau = \tau_f$, where $b(\tau_f) = 0$. According to [72], the tunneling probability is given by

$$P(\varphi) = \mathrm{e}^{-S(\varphi)+S_0}, \tag{B.3}$$

where $S(\varphi)$ is the Euclidean action for the tunneling trajectory $(\varphi(\tau), b(\tau))$, and $S_0 = S(\varphi_0)$ is the Euclidean action for the initial configuration $\varphi = \varphi_0$. $S(\varphi)$ in equation (B.3) for the tunneling probability is the integral over the whole instanton solution, rather that the integral over its half providing the tunneling amplitude.

The tunneling action is given by

$$S(\varphi) = \int \mathrm{d}^4x\sqrt{g}\left(-\frac{1}{2}R + \frac{1}{2}(\partial\varphi)^2 + V(\varphi)\right). \tag{B.4}$$

In $d = 4$ the trace of the Einstein equation is $R = (\partial\varphi)^2 + 4V(\varphi)$. Therefore the total action can be represented by an integral of $V(\varphi)$:

$$S(\varphi) = -\int \mathrm{d}^4x\sqrt{g}\,V(\varphi) = -2\pi^2\int_0^{\tau_f}\mathrm{d}\tau\, b^3(\tau)\, V(\varphi(\tau)). \tag{B.5}$$

The Euclidean action calculated for the false vacuum de Sitter solution $\varphi = \varphi_0$ is given by

$$S_0 = -\frac{24\pi^2}{V_0} < 0. \tag{B.6}$$

This action for de Sitter space S_0 has a simple sign-reversal relation to the entropy of de Sitter space $\mathbf{S_0}$ [73]:

$$\mathbf{S_0} = -S_0 = +\frac{24\pi^2}{V_0}. \tag{B.7}$$

Therefore the typical time of the tunneling from the de Sitter vacuum φ_0 to the de Sitter vacuum φ_1 given by $t_{\text{tunn}} \sim P^{-1}(\varphi)$ can be represented in the following way:

$$t_{\text{tunn}} = \mathrm{e}^{S(\varphi)+\mathbf{S_0}} = \mathrm{e}^{\frac{24\pi^2}{V_0}}\,\mathrm{e}^{S(\varphi)}. \tag{B.8}$$

Equation (B.5) implies that for the tunneling through the barrier with $V(\varphi) > 0$ the action $S(\varphi)$ is always negative, $S(\varphi) < 0$. This means that *the typical time of the tunneling from the de Sitter vacuum* φ_0 *to the de Sitter vacuum* φ_1 *is always finite and smaller than* $\mathrm{e}^{24\pi^2/V_0}$. This is very important because it allows tunneling between all de Sitter vacua separated by barriers with $V > 0$, even if the barriers are extremely high [55]. In other words, all such vacua become interlinked; one may start in one of them and then

Let us now assume that the lifetime of a de Sitter vacuum is much greater than the typical cosmological time $H^{-1} \sim V_0^{-1/2}$. This is a very mild assumption because the probability of tunneling is exponentially suppressed. Then the decay of the de Sitter state never completes in the whole universe. Indeed, the tunneling is a local process which occurs due to formation of bubbles of the new vacuum. During the exponentially large time required for the tunneling, the non-decayed part of the universe, outside of the bubbles of the new vacuum, continues expanding exponentially, which leads to eternal inflation. This is the same reason why inflation in the old inflationary scenario never ends [10], which was a major problem fully resolved only with the invention of the slow-roll chaotic inflation [15]. In the string theory landscape [57], the slow-roll inflation may begin *after* the tunneling, which is the standard way to solve the the graceful ending problem of the old inflation.

If the decay of some de Sitter states never completes because of the exponential expansion, the transition from these states to all other vacua of string theory becomes possible. As a result, the universe becomes a multiverse consisting of exponentially many exponentially large parts with different properties. A more detailed discussion of the tunneling process, including the transitions between different vacua in the situations where the Coleman–De Luccia instantons do not exist, can be found [55].

Finally, one should note that the transitions between different vacua is not the only way to implement the string landscape scenario. Even if for some particular reason the transitions between some specific string theory vacua are forbidden (e.g. if they are separated by some exotic barriers with $V < 0$), each of these vacua is still a part of a more general inflationary multiverse scenario based on quantum cosmology. Indeed, each of these vacua corresponds to a separate branch of the wave function of the universe, exactly in the same sense as separate universes with different constant values of fluxes $F_{\mu\nu\lambda\sigma}$ in the first anthropic solution of the cosmological constant problem, which was proposed long ago in my review of inflationary cosmology in *Reports on Progress in Physics* [38].

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