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THE ORIGIN AND EARLIEST RECEPTION OF BIG-BANG COSMOLOGY

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Abstract. The basic idea of big-bang cosmology is that the universe came into being a finite time ago and since then developed into its present state. More or less scientifically based versions of this idea can be found in the nineteenth century, but it was only after Einstein's general theory of relativity that it became possible to speak of a dynamical universe in a full sense. Although mathematical models of a big-bang universe were included in Friedmann's theory of 1922, the true beginning of (physical) big-bang cosmology should be dated to 1931, when Lemaître suggested his picture of the primordial state of the universe as a giant atomic nucleus. This paper outlines the origin of the big-bang hypothesis and pays particular attention to the works of Lemaître. It also deals with the earliest reception of the hypothesis, but only up to the mid-1930s.

1. PRE-RELATIVISTIC NOTIONS OF A WORLD OF FINITE AGE

The concept of a finite-age universe is not, of course, an invention of twentieth-century relativistic cosmology. But in so far it was discussed in earlier times, it was almost exclusively in a philosophical or religious context. Until about 1930 the universe at large was generally conceived as static, meaning that its temporal dimension was disregarded. Whereas it was admitted that the objects of the universe, such as stars and nebulae, developed over time, the large majority of astronomers and physicists paid no attention to a possible evolution of the universe as a whole. Still, arguments in favour of a universe that had existed only for a finite period of time, and hence could be ascribed a beginning or an origin, were not entirely missing in nineteenth and early-twentieth century science. There were at least three such arguments, all of them of a qualitative nature but nonetheless based on scientific reasoning.

As early as 1858 the noted German astronomer Johann Mädler pointed out that the notorious Olbers' paradox of the dark night sky could be circumvented if it was assumed that "the world is created, and hence is not eternal" (Tipler 1988). He reasoned that if the stars had existed only for a limited period of time, the light from those very far away would not yet have reached the Earth. A few other astronomers in the second half of the nineteenth century repeated or discussed the argument, but it was never taken very seriously. Another way to argue for a universe of finite age was by appealing to the second law of thermodynamics. Assuming that this law is applied to the entire universe, it apparently follows that universe cannot have existed eternally, for if so, the entropy would already have reached its maximum state: since we do not live in a world of very high entropy, the universe must have had a beginning. This so-called entropic creation argument dates back to the late 1860s and it continued to be discussed well into the twentieth century, if more by philosophers and theologians than by physical scientists (Kragh 2004, pp. 50-69; Neswald 2006, pp. 236-294). Some theologians and religious scientists used the argument apologetically and for this reason it became hotly debated in the cultural and ideological struggle in the late nineteenth century. Among those involved in the debate were thinkers as diverse as Ernst Mach, Ernst Haeckel, Friedrich Engels, Herbert Spencer and Friedrich Nietzsche, but their arguments relied only peripherally on scientific knowledge and for this reason I shall ignore them in the present context.

The entropic argument relies on a fundamental physical function that varies, in this case increases, monotonically in time. If other such functions or quantities exist, they may be used similarly, which is what happened after radioactivity had been discovered. A few scientists noticed that the irreversible phenomenon of radioactivity might serve as a cosmic arrow of time, thereby producing an additional argument against the eternity of the world. The counterfactual logic was basically the same: if the material universe had existed in an infinity of time, all radioactive substances would have vanished. Since there is in fact radioactive metals in the crust of the Earth – and presumably all over in the universe – the world must have had a beginning of a sort. This argument was first proposed by the Austrian physicist Arthur Erich Haas in 1911, but although mentioned a few times in the literature it played no great role in astronomy and cosmology (Kragh 2008). Among the few astronomers who did pay attention to it was Arthur Eddington, who in a lecture of 1922 on the relationship between geology and astronomy said as follows:

> *In radioactivity we see a mechanism running down which must at some time have been wound up. Without entering into any details, it would seem clear that the winding-up process must have occurred under physical conditions vastly different from those in which we now observe only a running-down.*

However, Eddington did not identify these vastly different physical conditions with a primeval state of the universe, but with "the general brewing of material which occurs under the intense heat in the interior of the stars" (Eddington 1923). He was at the time engaged in an attempt to explain stellar energy production by means of nuclear processes, either proton-electron annihilation or the synthesis of four hydrogen nuclei into one helium nucleus.

None of the three arguments here mentioned were received favourably by physicists and astronomers, the large majority of whom simply ignored the question of a possible origin of the universe. Until the 1930s this was a non-problem, something which might be left to philosophers and theologians to discuss. I know of only a handful of physical scientists who commented on the issue before it was turned into a scientific hypothesis by Lemaître in 1931. One of them was the British astrophysicist Herbert Dingle, who in a popular book of 1924 suggested that the nebular redshifts observed by Melvin

Slipher might indicate "the legacy of a huge disruption, in the childhood of matter, of a single parent mass" (Dingle 1924, p. 399). But Dingle did not take the suggestion seriously and elsewhere in his book he affirmed the eternity of the universe.

2. THE DYNAMICAL MODELS OF FRIEDMANN AND LEMAÎTRE

Einstein's 1917 application of the general theory of relativity to the universe marks a watershed in the history of cosmology. By describing physical space as positively curved, he made it possible to speak of a spatially finite yet unbounded universe. He also made it possible to speak of a radius of curvature varying in time, but it is well known that he ignored this possibility and maintained that the universe was static. It was basically for this reason that he introduced the cosmological constant as the characteristic term in a new kind of anti-gravitational force. Indeed, until 1922 it was taken for granted that the universe, as described by Einstein's cosmological field equations, was static. Contrary to the modern understanding, this was also the case with the empty model proposed by Willem de Sitter in 1917 as an alternative to the Einstein model.

Although Alexander Friedmann's important paper of that year, published in volume 10 of the *Zeitschrift für Physik*, is today recognized as a groundbreaking contribution to cosmology, at the time it was neither understood nor well known. What matters is that Friedmann offered a complete and systematic analysis of the solutions of the Einstein equations that satisfy the cosmological principle. As a novelty, these included non-static solutions. By integrating what is today known as the Friedmann equations for closed models, he found a class of homogeneously expanding world models. As to the scale factor R, a measure of the size of the universe, he wrote: "Since R cannot be negative, there must be, as one decreases the time, a time when R vanishes ... a beginning of the world." Moreover: "The time since the creation of the world is the time which has passed from the moment at which space was [concentrated at] a point ($R = 0$) to the present state ($R = R_0$)" (Friedmann 1922). One may say that Friedmann introduced expanding as well as big-bang models in the mathematical sense, but it is important to recognize that it was in this sense only. His two papers of 1922 and 1924 – in which he discussed hyperbolic models of constant negative curvature – were thoroughly mathematical, whereas he showed almost no interest at all in the physical and astronomical aspects of the universe. It is only with hindsight he can be considered the discoverer of the expanding universe, and likewise it is to read too much into his paper if his comments are taken as support of a universe with a beginning in time (Kragh and Smith 2003). To put it briefly: Friedmann was not really interested in *the* universe, but only in mathematical models of the universe.

One kind of astronomical observation that Friedmann did not mention (although it was known to him in 1924) was the redshifts of the spiral nebulae that Slipher had first detected in 1912. It was not originally thought that these data were of cosmological significance, but during the 1920s it became increasingly clear that this was the case. In his seminal paper of 1927, Georges Lemaître included a detailed discussion of the known redshifts and connected them for the first time with dynamical models of the universe (Lemaître 1927). Not knowing of Friedmann's paper at the time, the Belgian physicist duplicated much of the mathematics found in the former's work. His differential equations were almost exactly as Friedmann had formulated them, except that Lemaître included a pressure term. Guided by the data of extragalactic redshifts, he chose to focus on the expanding solution that agreed best with observations. His favoured model was a closed universe expanding from an Einstein state, the size of which he estimated from observational data to be about 270 Mpc. Moreover, he found theoretically an approximately linear relationship between the recession velocity and the distance of the nebulae. "The receding velocities of extragalactic nebulae are a cosmical effect of the expansion of the universe," as he emphasized. For the expansion constant – later known as the Hubble constant – he obtained a value of about 625 km/s/Mpc, of the same order of magnitude that Hubble found observationally two years later.

Lemaître's model of 1927 did not belong to the big-bang class, as time could be traced back in cosmic history indefinitely, if only logarithmically. On the other hand, he seems to have conceived the static Einstein universe as a kind of pre-universe out of which the expansion had grown as the result of an instability. This instability might have been caused by the pressure of radiation, he suggested. At the end of his paper, he wrote: "In a static universe light emitted by matter travels round space, comes back to its starting point, and accumulates indefinitely. It seems that this may be the origin of the velocity of expansion R'/R which ... in our interpretation is observed as the radial velocity of extragalactic nebulae." The papers of Friedmann and Lemaître had in common that they were both ignored by contemporary astronomers and physicists, if for different reasons. Whereas a few scientists referred to Friedmann's article in the 1920s, no-one seems to have cited Lemaître's until it was rediscovered in 1930. There is little doubt that a main reason for this was Lemaître's unhappy decision to publish it in French and in the somewhat obscure *Annales de Société Scientifiques de Bruxelles*. But there were other reasons as well. Thus, it is known that he sent copies of the paper to both Eddington and de Sitter, who however failed to study it. Einstein, too, was acquainted with the main points of his theory, which Lemaître communicated to him during a conversation they had in Brussels in the fall of 1927. Einstein flatly denied that the young Belgian's theory described the real universe and as late as 1929 he reconfirmed in an article in *Encyclopedia Britannica* the static nature of the universe: "Nothing certain is known of what the properties of the space-time continuum may be as a whole," he stated. "Through the general theory of relativity, however, the view that the continuum is infinite in its time-like extent but finite in its space-like extent has gained in probability" (Einstein 1929, p. 108).

3. LEMAÎTRE'S PRIMEVAL-ATOM HYPOTHESIS

In May 1931 Lemaître published a brief note in *Nature*, in which he advocated the idea of a universe originating from an explosion of a supercompact, primordial entity of the form of an atomic nucleus. It is not known precisely what motivated him to suggest this very first version of a physical big-bang model, but we can get some insight by looking at an article he wrote a few months earlier, at a time when he still explored the meaning and consequences of the model based on his paper of 1927 (Kragh and Lambert 2007). In this earlier article of 1931, Lemaître introduced the idea of a "stagnation phase" due to a diminution of pressure and with the effect of starting the expansion from the Einstein state. At that time he seems to have visualized the initial Einstein universe as a kind of primeval gas: "If, in a universe of equilibrium, the pressure begins to vary, the radius of the universe varies in the opposite sense. Therefore, stagnation processes include expansion." Considering the effect of a sudden stagnation process in which the pressure dropped instantaneously to zero, he found that "the epoch of the rupture of equilibrium" would have taken place some ten to hundred billion years ago. If the pressure completely dominated over matter, this would imply that "all the energy was in the form of electromagnetic radiation and suddenly condensed into matter" (Lemaître 1931a). That is, Lemaître imagined a static proto-universe made up of radiation energy which suddenly, at $t = 0$, underwent a gigantic materialization. This was a picture that had some similarity with James Jeans' contemporary idea of matter being originally created as protons and electrons out of high-energy photons (Jeans 1928).

It is less well known that Lemaître was also inspired by Robert Millikan's somewhat controversial ideas regarding the nature of the cosmic rays which he – Millikan – believed were high-energy photons arising from nuclear-building-up processes in the depths of space. Although Millikan used this idea as an argument in favour of an eternal and cyclic universe, i.e. the very opposite of the big-bang universe, Lemaître found the idea interesting and in 1930 he reconsidered it in a different cosmological perspective. "One could concede that the light had been the original state of matter," he wrote, "and that all the matter condensed in the stars was formed by the process proposed by Millikan" (Lemaître 1930). In a letter he wrote to de Sitter at about the same time he continued to speculate about the cosmic role of electromagnetic radiation. He said that "there is almost no means to avoid the conclusion that . . . the radiation emitted by the stars recrystallizes into matter and gives birth to the cosmic rays" (Luminet 1997, p. 305). The hypothetical light-to-matter crystallization process was in agreement with Millikan's view, but Lemaître did not share Millikan's conviction that the cosmic rays were made up of photons. On the contrary, he soon came to the conclusion that they were charged particles, the remnants of the original explosion of the primordial atom.

One can perhaps reconstruct Lemaître's way of thinking in the crucial phase between the summer of 1930 and the spring of 1931 by admitting that he initially thought of the original state of the universe in terms of light or photons. At some stage, probably in early 1931, he realized that a primeval universe of light would not do and he consequently changed the imagery from light to atoms. The change was facilitated by the light-matter analogy associated with quantum theory, that is, by considering light as photons that might crystallize into material particles. The way in which he spoke of the disintegration of the primeval atom after May 1931 suggests that the conceptual origin of the idea is to be found in the abandoned hypothesis of a universe initially made up of light. Moreover, he most likely associated this idea with a religious dimension, for as a young man he had speculated about light as a unifying concept of all physical phenomena in accordance with the words of the Bible (Lambert 1997). Although he did not pursue this biblical interpretation, he maintained an interest in the possible cosmological significance of primeval light.

Lemaître proposed his novel picture of the beginning of the universe in a brief and most remarkable article of 9 May 1931, a purely qualitative argument of less than 500 words. This is not the place to analyze this article in detail, but what should be noted is that his argument for a beginning in an undifferentiated "primeval atom" was rooted in the principles of quantum mechanics and thermodynamics. In fact, it was the first time that these two fundamental sciences were brought together in a cosmological context. As to thermodynamics:

> *Thermodynamical principles from the point of view of quantum theory may be stated as follows: (1) Energy of constant total amount is distributed in discrete quanta. (2) The number of distinct quanta is ever increasing. If we go back in the course of time we must find fewer and fewer quanta, until we find all the energy of the universe packed up in a few or even in a unique quantum.*

As to the quantum theory:

> Clearly the initial quantum could not conceal in itself the whole cause of evolution, but, according to the principle of indeterminacy, that is not necessary. Our world is now understood to be a world where something really happens; the whole story of the world need not to have been written down in the first quantum like the song on the disc of a phonograph. The whole matter of the world must have been present at the beginning, but the story it has to tell may be written step by step.

Although Lemaître did not refer explicitly to entropy in his note to nature of 1931, he did make use of a form of the entropic creation argument, only did he formulate it in a different and not very clear way. In fact, his argument is objectionable not only because it did not take into account the expansion of space but also because it applied radiation thermodynamics as if it was valid for the disintegrating primordial world (Kragh and Lambert 2007).

What kind of entity was the original quantum? Lemaître boldly suggested that it might be likened to a huge atomic nucleus with an exceedingly large atomic number. "We could conceive," he wrote, "the beginning of the universe in the form of a unique quantum, the atomic weight of which is the total mass of the universe. This highly unstable atom would divide into smaller and smaller atoms by a kind of super-radioactive process." One should not pay attention to his reference to "atom" rather than "atomic nucleus" – the point was that the original state was to be understood as an uncomposite (non-composite?) body, as something completely undifferentiated and devoid of physical properties.

Radioactivity thus played an important role in Lemaître's scenario, but only as a mechanism that secured an acausal disintegration of the primeval atom. He did not mention another aspect of radioactivity, namely the argument based on the existence of radioactive substances in the crust of the Earth and elsewhere in the universe. Although the argument remained unmentioned in his letter to *Nature*, we know that he was aware of it and that it stimulated his thinking about a world of finite age. The half-lives of radioactive elements such as thorium and uranium were known to be of the same order of magnitude as the inverse Hubble parameter (a few billion years) which, according to the cosmology of the expanding universe, is a rough measure of the age of the universe. Could this, Lemaître asked himself, be just a coincidence? Did it not indicate that our present world is the nearly burned-out result of a previous highly radioactive universe? In a paper of 1949 Lemaître told (wrote) that the argument was indeed a motivating factor for his proposal of the primeval-atom model:

> *The idea of this hypothesis arose when it was noticed that natural radioactivity is a physical process which disappears gradually and which can, therefore, be expected to have been more important in earlier times. If it were not for a few elements of average lifetimes comparable to T_H [the Hubble time], natural radioactivity would be completely extinct now. It might be thought, therefore, that radioactive elements did exist which are actually transformed into stable elements. (Lemaître 1949, p. 452)*

Lemaître presented a better argued and more comprehensive version of his explosion theory later in 1931, in an article in *Revue des Questions Scientifiques* and also in an address to the British Association for the Advancement of Science. In this version he stressed the role of the stagnation phase made possible by the assumption of a positive cosmological constant. As he pointed out, there was a double bonus from introducing the stagnation phase: for one thing, it stretched the timescale and thus provided a solution to the age paradox, namely that the age of the stars was larger than the age of the universe as inferred from the Hubble parameter; for another thing, it made it easier to explain how galaxies were formed in the early universe. Contrary to Einstein, Lemaître was a great advocate of the cosmological constant and he continued to believe in it until his death in 1966. Most other cosmologists disagreed, but more recent events in observational cosmology – the discovery of the accelerated expansion at the end of the 1990s – have justified his view.

Strictly speaking, Lemaître's model of 1931 was not really of the big-bang type, as it did not include a point-like universe (the singularity $R = 0$) for $t = 0$. Always adopting a physical (rather than mathematical) point of view, he resisted the idea of an initial singularity of infinite mass density. According to Lemaître's thinking, at $t = 0$ the universe had already "existed" in the shape of the material primeval atom that contained within it the entire mass of the universe, and the radius of which he estimated to be a few astronomical units. The matter density would correspond to that of an atomic nucleus, the highest density known at the time. His primeval atom was simple in an absolute, almost metaphysical sense – inaccessible to scientific inquiry, devoid of physical properties, and hence non-existent from a physical point of view. While he originally spoke of the initial state as a unique atom, he later likened it to a gigantic "isotope of the neutron." In a certain sense the primeval atom existed before its radioactive explosion, but there was no way to tell for how long as time had not yet any meaning. In another sense one may say that the primeval atom came close to being a physical metaphor for nothingness. This was apparently how Eddington looked at it, when he said: "To my mind undifferentiated sameness and nothingness cannot be distinguished philosophically" (Eddington 1933, p. 57).

4. SOME EARLY RESPONSES TO THE EXPLOSION (EXPLODING) UNIVERSE

From a sociological point of view, Lemaître's theory of the exploding universe was no success at all. In fact, for a year or two it went unnoticed in the astronomical literature, possibly because the *Revue des Questions Scientifiques* was not known by the majority of astronomers. The first responses were either highly critical or even attempted to ridicule the hypothesis which was considered too fanciful to count as genuine science. According to John Plaskett, a Canadian astronomer, Lemaître's explosion theory was "the wildest speculation of all [cosmological hypotheses] ... an example of speculation run mad without a shred of evidence to support it" (Plaskett 1933, p. 252).

Viewed as a mathematical solution to the Friedmann equations, Lemaître's model was of course acceptable, but there was a general opposition against interpreting it realistically, that is, as a description of how our universe has actually evolved. A universe with a beginning in time was considered very strange, especially as long as the hypothesis was not solidly supported by observational evidence. In 1931, Lemaître suggested that the cosmic rays constituted such evidence for a radioactive origin of the world, but his idea was not generally accepted. Among those who were opposed, either directly or indirectly, towards the primeval atom hypothesis were leading experts such as Howard Robertson, Richard Tolman and Eddington. It is characteristic that when Robertson in early 1933 published his influential review of relativistic cosmology, he included in its extensive bibliography only Lemaître's publications on the expanding Lemaître-Eddington universe and none of his works on the primeval atom universe. Robertson found this kind of big-bang solution to be contrived and unappealing (Robertson 1933). The American physicist Paul Epstein, who had done important work in the old (Bohr-Sommerfeld) quantum theory, published in 1934 another review of the expanding universe. Although he briefly mentioned Lemaître's model in the text, and welcomed it because it retained the cosmological constant, he did not refer to any of the publications of the Belgian cosmologist (Epstein 1934).

Einstein's position in the earliest phase of big-bang cosmology deserves to be briefly mentioned. In 1931 he discussed a model with big-bang features, independently and apparently without knowledge of Lemaître's new hypothesis (Einstein 1931). The closed model was of the big-bang type in so far it formally included $R = 0$ not only for $t = 0$ but also for the time at which the cycle had been completed in a "big squeeze" (this name seems to have been introduced by Gamow in 1952). The 1931 model is often referred to as cyclic or oscillatory, but in fact Einstein considered only a single cycle. The following year he collaborated with de Sitter in suggesting a different cosmological model in which there was no space curvature, no pressure and no cosmological constant. The later so well known Einstein-de Sitter model of a universe of critical density was obviously of the big-bang type and it followed from it that the age of the universe was given by 2/3 of the Hubble time. However, Einstein and de Sitter did not mention this and neither did they refer to Lemaître's work although at the time they knew about it (Einstein and de Sitter 1932). Nonetheless, Einstein was not opposed to the primeval atom hypothesis and in conversations with

Lemaître in 1932-33 he gave it his unofficial support.

I shall not here deal (here) with the further development of relativistic big-bang theory, which I have given an account of elsewhere (Kragh 1996). Yet it should be pointed out that after World War II, at a time when Lemaître's hypothesis enjoyed but little support, it was independently revived and developed along the lines of nuclear physics by George Gamow and his collaborators in the United States. This nuclear-physical version of the early universe led to several promising results, but nevertheless died out after a few years of research. Remarkably, in the decade following 1953 big-bang cosmology was nearly non-existent. It was only from about 1965, with the discovery of the cosmic microwave background and the demise of the steady-state theory, that the big-bang theory was developed into a viable state which soon became generally accepted as a kind of standard theory. As far as paternity is concerned, we may thus say that big-bang cosmology had four fathers, each of them belonging to different periods. The two first fathers were Friedmann and Lemaître, and the third was Gamow. Several physicists were engaged in the development in the mid-1960s that led to the hot big-bang standard theory, the most prominent of them being Robert Dicke, James Peebles and Yakov Zel'dovich. The genealogy of modern big-bang theory is as complex as it is interesting.

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